education analytics

LINKING STUDY BETWEEN SOUTH CAROLINA COLLEGE- AND CAREER-READY ASSESSMENT (SC READY) AND NWEA MAP GROWTH ASSESSMENT, GRADES 3-8

- February 2024

110 E Main Street, Ste. 1000
Madison, WI 53703

608.466.4966

edanalytics.org

CONTENTS

INTRODUCTION

In the spring of 2023, the South Carolina Education Oversight Committee (SC EOC), in collaboration with the South Carolina Department of Education (SCDE), partnered with Education Analytics (EA) to complete a linking study between the South Carolina College- and Career-Ready Assessment (SC READY) in Mathematics and English Language Arts (ELA) and the NWEA MAP Growth assessment in Mathematics and Reading, respectively. This report outlines the methodology used by EA and the outcomes of the linking study. The goal of this study is to statistically connect the SC READY and MAP assessments' scale scores in grades 3-8 to facilitate further comparisons of proficiency status on these two assessments.

METHODS

Data

This linking study used data from the SC READY and MAP Mathematics and ELA/Reading assessments administered in Spring 2023.[1] Students were matched through their state IDs or district IDs. Only matched students who took the MAP assessments within 30 days of SC READY[2] in Spring 2023 were included in this study.

Post-Stratification Weighting

To increase the generalizability of the linking results based on the matched student sample to South Carolina's student population, EA applied post-stratification weights to the calculations. The variables used in the weighting process include gender, race/ethnicity, English learner (EL) status, poverty status, disability status, and whether a student met or exceeded standards on the same subject SC READY assessment. Through post-stratification weighting, the weighted study sample provides a closer match with the South Carolina state population on these key demographic and academic performance variables than the original sample.

Raking was used to calculate the post-stratification weights. Raking involves an iterative proportional fitting procedure, which introduces each demographic and academic variable in a

[1] EA also explored data from Spring 2021 and Spring 2022 but agrees with SC EOC and SCDE that linking results from Spring 2023 are preferred given they are the furthest from COVID impacts and the most recently available data. The research sample sizes from 2023 are sufficient and the model diagnostics are good; therefore, linking results from Spring 2023 are reported.

[2] The SC READY data do not include the actual test administration dates, so this is an approximation based on SCDE's 2022–23 Assessment Schedule.

sequence so that it ensures the sample accurately represents the population of all variables under consideration. The variables are introduced one at a time, which allows for the incorporation of more variables in the weighting procedure. The raking procedure includes the following steps:

1. Collect marginal distributions of each weighting variable from South Carolina's student population.
2. Calculate marginal distributions of each weighting variable from the matched sample.
3. Calibrate post-stratification weights using the raking procedure.
4. Trim the weight to be within the range of 0.3 and 3. This is done to minimize the impact of outlier cases which may carry extremely large or small weights.
5. Apply the weights to the matched sample before conducting the linking analyses.

Equipercentile Linking

The linking analyses between SC READY and MAP assessments were conducted using the equipercentile linking method (Kolen & Brennan, 2014). The equipercentile linking function is determined by the cumulative distribution functions of the two assessments. In the linking process, the cumulative distribution function of scores on the spring MAP assessment converted to the SC READY score scale is aligned to the cumulative distribution function of scores on SC READY. More specifically, this process utilizes percentile ranks, which indicates the percentage of scores in the frequency distribution that fall below a particular score. Equipercentile linking then establishes the relationship between the two sets of test scores by identifying corresponding percentile ranks of the test scores. Thus, we can establish scores on the spring MAP assessment that are aligned to the three SC READY achievement level cut scores (i.e., cut score between Does Not Meet Expectations and Approaches Expectations, cut score between Approaches Expectations and Meets Expectations, and cut score between Meets Expectations and Exceeds Expectations) for mathematics and ELA at grades 3-8. The linking function can be written as:

$$e_Y(x) = G^{-1}[F(x)]$$

where x represent a score on test X (e.g., SC READY ELA), $e_Y(x)$ is its corresponding score on test Y (e.g., MAP Reading), $F(x)$ is the cumulative distribution function of a given score on SC READY, and G^{-1} is the inverse of the cumulative distribution function for MAP, which indicates the MAP scale score corresponding to a given percentile in the distribution.

Prior to the equipercentile linking, the polynomial log-linear pre-smoothing method is applied to reduce irregularities of the test score distributions. This method fits polynomial functions to the log of the sample density to smooth the distributions of the assessments (Holland & Thayer, 1987, 2000; Rosenbaum & Thayer, 1987).

Extending from Spring to Fall and Winter

To support the needs of SC EOC and SCDE to extend linked MAP test scores from spring to the fall and winter terms, EA also estimated scores needed to meet expectations of the SC READY test in the fall and winter terms prior to the spring term in grades 3-8. This was done by calculating the mean MAP scores in each term, subject, and grade in 2022-23 among all SC students who took the MAP test. The average change in scores between fall and spring, and winter and spring were subtracted from the spring cut scores determined by the linking analyses. These fall and winter cut scores are reported along with spring cut scores in the results section.

Classification Accuracy

Classification accuracy statistics are used to evaluate the degree to which the equivalent scores on the spring MAP assessment to the SC READY achievement level cut scores can be used to accurately classify students' proficiency status. In this report, we summarize seven types of commonly used classification accuracy statistics (see Table 1) based on the cut score between Approaches Expectations (i.e., not proficient) and Meets Expectations (i.e., proficient).

To facilitate appropriate interpretations of the linking results, a bootstrap analysis was also conducted whereby each linking analysis was replicated 1,000 times through iterative resampling of each study sample with replacement. The bootstrap standard errors help us understand the amount of error associated with the estimates. The bootstrap standard errors associated with the test cut scores are reported in Table 10.

Table 1. Description of Classification Accuracy Summary Statistics

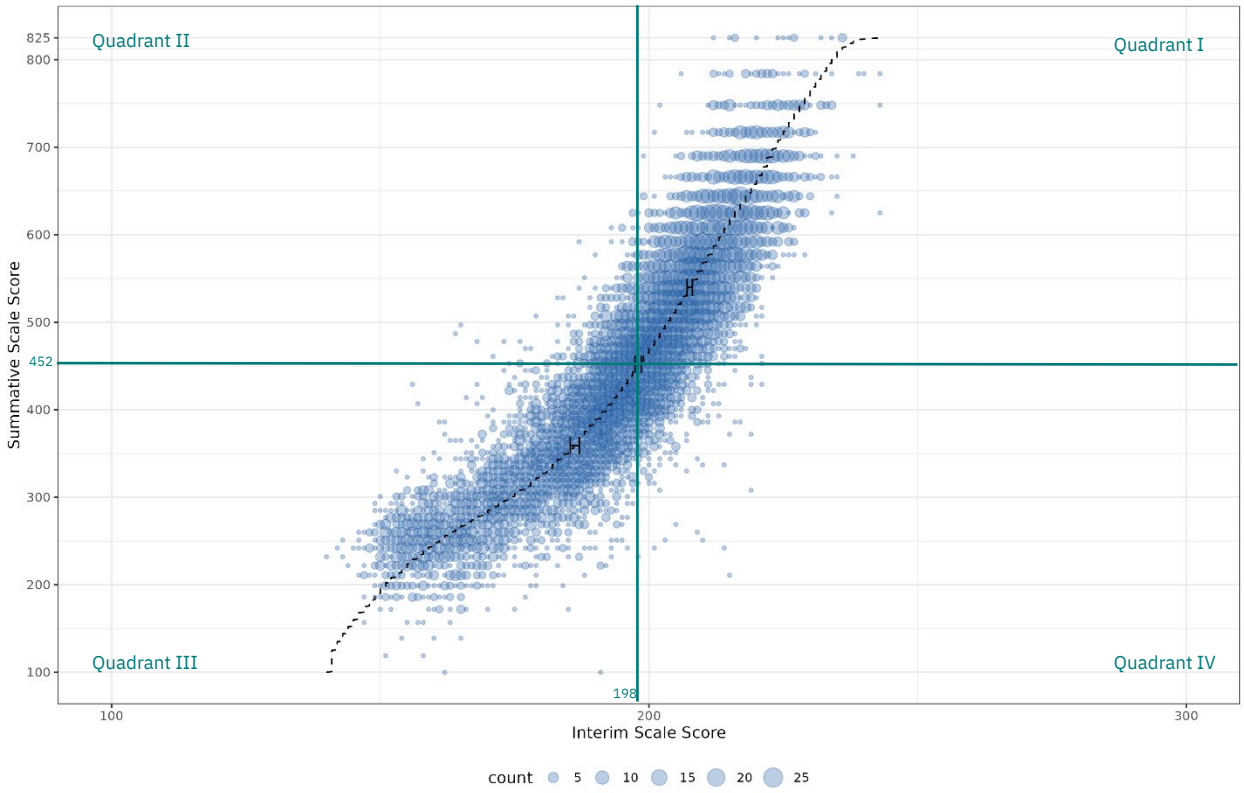
Statistic	Description
Overall Classification Accuracy	Proportion of the study sample with correct proficiency classifications on SC READY based on MAP cut scores. Calculated as $(TP+TN)/\text{Total Sample Size}$
False Positive (FP) Rate	Proportion of proficient students based on MAP cut scores among those observed as not proficient on the SC READY test. Calculated as $FP/(FP+TN)$
False Negative (FN) Rate	Proportion of students who were not proficient based on MAP cut scores among those observed as proficient on the SC READY test. Calculated as $FN/(FN+TP)$
Sensitivity	Proportion of proficient students based on MAP cut scores among those observed as proficient on the SC READY test. Calculated as $TP/(TP+FN)$
Specificity	Proportion of students who were not proficient based on MAP cut scores among those observed as not proficient on the SC READY test. Calculated as $TN/(TN+FP)$
Precision	Proportion of observed proficient students on the SC READY test among those classified as proficient based on MAP cut scores. Calculated as $TP/(TP+FP)$
Area Under the Curve (AUC)	An overall indication of the diagnostic accuracy of a Receiver Operating Characteristic (ROC) curve. AUC tells us how well the MAP cut score separates the study sample as proficient and not proficient in accordance with the SC READY ELA test cut score. An AUC above 0.80 is considered "convincing evidence" of classification accuracy.

Note: TP = true positive; TN = true negative; FP = false positive; FN = false negative.

Figure 1 is a scatterplot of the SC READY ELA and MAP Reading scores from grade 3 in Spring 2023. The best-fitting curve (i.e., the black dashed line) shows the MAP Reading scores that correspond to the SC READY ELA scores through the linking estimation. For example, the SC READY ELA score of 452 is the cut score for "Meets Expectations" at grade 3. This score corresponds to the MAP Reading score of 198 with a standard error of 0.29 in the linking results. The narrow black bands plotted around the dashed curve shows the 95% confidence interval. The small standard errors provide evidence of the accuracy of the linking model. However, the SC READY ELA score of 452 and the MAP Reading score of 198 should not be used interchangeably. As shown in Figure 1, not all students who scored 198 and above on the MAP Reading test also scored 452 or higher on the SC READY ELA test in Spring 2023. Specifically, students in Quadrant IV scored lower than 452. Similarly, students who met or exceeded expectations (i.e., scored 452 or above) on the SC READY ELA test had a wide range

of scores on the MAP Reading test, some of which were below 198 (i.e., students in Quadrant II). We recommend users examine the scatterplot of observed test scores and bootstrap standard errors to gain a more complete understanding of the linking results and associated limitations.

Figure 1. Scatterplot of the SC READY ELA and MAP Reading Scores, Grade 3, Spring 2023



RESULTS

Study Sample

The linking study sample includes students who took both the SC READY and MAP Mathematics and ELA/Reading assessments within 30 days in Spring 2023 from 39 school districts in South Carolina. Tables 2 and 3 summarize the sample characteristics, including student demographic subgroups (i.e., gender, race/ethnicity, poverty, EL, and disability status) and percent of students who met or exceeded standards on the SC READY Mathematics and ELA assessments at each grade in the original sample before post-stratification weighting.

Table 2. Unweighted Linking Study Sample Characteristics: Mathematics

Subgroup	Percent of Students by Grade					
	3	4	5	6	7	8
Female	48.3	49.0	48.9	48.4	49.9	49.9
Male	51.7	51.0	51.1	51.6	50.1	50.1
Black	32.6	32.5	33.2	32.2	33.3	33.0
Hispanic	9.8	10.0	10.2	11.2	10.8	11.1
White	48.6	49.2	48.2	49.0	48.3	48.7
Other	9.0	8.3	8.4	7.6	7.7	7.2
Pupil in Poverty	66.8	66.8	64.8	61.7	61.6	60.0
English Learner	8.3	6.2	6.3	7.4	7.7	7.0
Student with Disabilities	16.7	16.2	14.2	14.0	12.9	11.9
SC READY: Meets Expectations or Exceeds Expectations	53.5	48.0	45.6	37.5	31.8	33.5
SC READY: Does Not Meet Expectations or Approaches Expectations	46.5	52.0	54.4	62.5	68.2	66.5

Table 3. Unweighted Linking Study Sample Characteristics: ELA

Subgroup	Percent of Students by Grade					
	3	4	5	6	7	8
Female	48.9	49.0	49.5	48.2	49.4	49.7
Male	51.2	51.0	50.5	51.8	50.6	50.3
Black	37.4	32.8	33.9	33.6	34.7	34.6
Hispanic	11.0	9.9	10.5	11.2	10.8	11.2
White	42.8	48.8	47.3	47.3	46.5	47.0
Other	8.8	8.5	8.2	7.8	8.0	7.3
Pupil in Poverty	66.6	66.4	66.5	63.4	63.4	61.9
English Learner	9.0	6.4	6.5	7.4	7.6	7.0
Student with Disabilities	16.8	16.4	14.5	14.2	13.3	12.2
SC READY: Meets Expectations or Exceeds Expectations	50.9	56.5	54.7	51.2	49.2	52.3
SC READY: Does Not Meet Expectations or Approaches Expectations	49.1	43.5	45.3	48.8	50.8	47.7

Distributions of the weighting variables in the South Carolina student population are listed in Table 4. After adjusting for post-stratification weights, the sample characteristics were recalculated. They are shown in Tables 5 and 6 at each grade level for mathematics and ELA, respectively. After weighting, the sample distributions are almost identical to the population distributions.

Table 4. South Carolina Student Population Characteristics

Subgroup	Percent of Students by Grade					
	3	4	5	6	7	8
Female	48.8	49.1	49.1	48.9	49.2	49.4
Male	51.2	50.9	50.9	51.1	50.8	50.6
Black	30.3	30.7	31.0	31.5	32.0	32.0
Hispanic	12.7	12.6	12.9	12.9	13.2	12.6
White	48.3	48.1	47.9	47.7	47.2	48.0
Others	8.7	8.6	8.3	7.9	7.7	7.4
Pupil in Poverty	62.8	63.0	62.4	62.0	61.8	61.0
English Learner	11.4	8.6	8.7	9.0	9.7	8.7
Student with Disabilities	16.7	15.9	14.9	14.3	13.9	12.9
SC READY Math: Meets Expectations or Exceeds Expectations	53.6	47.0	44.7	36.6	31.0	31.6
SC READY Math: Does Not Meet Expectations or Approaches Expectations	46.4	53.0	55.3	63.4	69.0	68.4
SC READY ELA: Meets Expectations or Exceeds Expectations	53.4	57.1	55.2	53.4	50.0	53.1
SC READY ELA: Does Not Meet Expectations or Approaches Expectations	46.6	42.9	44.8	46.6	50.0	46.9

Source: https://ed.sc.gov/data/test-scores/state-assessments/sc-ready/2023/state-scores-by-grade-level-and-demographic/?districtCode=9999&schoolCode=1001

Note: Information in this table is based on students who took the 2023 SC READY Mathematics and ELA statewide tests. In the few cases where students' race/ethnicity and poverty status differ by 0.1%, numbers shown are the average of percentages from mathematics and ELA.

Table 5. Weighted Linking Study Sample Characteristics: Mathematics

Subgroup	Percent of Students by Grade					
	3	4	5	6	7	8
Female	48.8	49.1	49.1	48.9	49.2	49.4
Male	51.2	50.9	50.9	51.1	50.8	50.6
Black	30.2	30.7	31.0	31.4	31.9	32.0
Hispanic	12.8	12.6	12.9	13.0	13.2	12.7
White	48.3	48.1	47.9	47.7	47.2	47.9
Other	8.7	8.6	8.3	7.9	7.7	7.4
Pupil in Poverty	62.8	63.0	62.4	62.0	61.8	61.0
English Learner	11.4	8.6	8.7	9.0	9.7	8.7
Student with Disabilities	16.7	15.9	14.9	14.3	13.9	12.9
SC READY: Meets Expectations or Exceeds Expectations	53.6	47.0	44.7	36.6	31.0	31.6
SC READY: Does Not Meet Expectations or Approaches Expectations	46.4	53.0	55.3	63.4	69.0	68.4

Table 6. Weighted Linking Study Sample Characteristics: ELA

Subgroup	Percent of Students by Grade					
	3	4	5	6	7	8
Female	48.8	49.1	49.1	48.9	49.2	49.4
Male	51.2	50.9	50.9	51.1	50.8	50.6
Black	30.3	30.7	31.0	31.4	31.9	32.0
Hispanic	12.8	12.6	12.9	12.9	13.2	12.7
White	48.3	48.1	47.9	47.7	47.2	47.9
Other	8.7	8.6	8.3	7.9	7.7	7.4
Pupil in Poverty	62.8	63.0	62.4	62.0	61.7	61.0
English Learner	11.4	8.6	8.7	9.0	9.7	8.7
Student with Disabilities	16.7	15.9	14.9	14.3	13.9	12.9
SC READY: Meets Expectations or Exceeds Expectations	53.4	57.1	55.2	53.4	50.0	53.1
SC READY: Does Not Meet Expectations or Approaches Expectations	46.6	42.9	44.8	46.6	50.0	46.9

Descriptive Statistics of Test Scores

Table 7 presents summary statistics of the SC READY and MAP Mathematics and ELA/Reading scores using the unweighted linking sample, which include the sample size, mean and standard deviation, and correlation (r) between the tests at each grade level. The correlations range from 0.83 (grade 7, ELA) to 0.88 (grade 3, ELA), which indicate moderate to strong associations between the two tests. This provides a good foundation for conducting a linking study between the SC Ready and MAP Mathematics and ELA/Reading tests.

Table 7. Descriptive Statistics of SC READY and MAP Mathematics and ELA/Reading Scores

		Grade					
		3	**4**	**5**	**6**	**7**	**8**
		Mathematics					
	N	8406	8394	8452	9212	9352	9655
	r	0.84	0.85	0.85	0.85	0.85	0.84
SC READY	Mean	457.9	489.2	536.8	522.1	547.4	582.9
	S.D.	115.1	119.0	105.9	106.4	106.2	106.8
	Min.	100.0	236.0	275.0	281.0	100.0	100.0
	Max.	825.0	850.0	875.0	900.0	925.0	950.0
MAP	Mean	200.1	209.1	216.8	218.9	223.4	228.5
	S.D.	14.8	15.8	16.9	17.2	18.9	19.4
	Min.	128.0	133.0	127.0	158.0	153.0	157.0
	Max.	246.0	268.0	278.0	283.0	290.0	319.0
		ELA					
	N	8979	7689	7634	9118	9283	9542
	r	0.88	0.85	0.85	0.86	0.83	0.84
SC READY	Mean	452.2	533.0	573.5	580.4	615.4	650.4
	S.D.	129.6	121.4	122.1	128.7	123.4	124.4
	Min.	100.0	230.0	199.0	224.0	280.0	316.0
	Max.	825.0	850.0	875.0	900.0	925.0	950.0
MAP	Mean	195.4	204.6	210.7	213.2	215.9	220.1
	S.D.	17.8	16.6	15.8	16.0	16.4	16.1
	Min.	140.0	140.0	144.0	159.0	157.0	160.0
	Max.	243.0	254.0	267.0	258.0	277.0	268.0

SC READY and MAP Cut-Score Equivalents

Tables 8 and 9 present the linking results between SC READY and MAP spring tests for mathematics and ELA, respectively. The top panel shows the ranges of SC READY scale scores at each proficiency level and grade level in 2022-23. The bottom panel shows the corresponding MAP scores.

Table 8. SC READY and MAP Cut Score Equivalents (Spring): Mathematics

Grade	SC READY			
	Does Not Meet Expectations	**Approaches Expectations**	**Meets Expectations**	**Exceeds Expectations**
3	100–359	360-437	438-542	543-825
4	100-400	401-480	481-562	563-850
5	100-447	448-534	535-621	622-875
6	100-452	453-542	543-626	627-900
7	100-487	488-576	577-648	649-925
8	100-526	527-614	615-682	683-950
	NWEA MAP			
	Does Not Meet Expectations	**Approaches Expectations**	**Meets Expectations**	**Exceeds Expectations**
3	100-189	190-199	200-210	211-350
4	100-200	201-210	211-218	219-350
5	100-203	204-218	219-230	231-350
6	100-209	210-224	225-235	236-350
7	100-215	216-231	232-241	242-350
8	100-220	221-236	237-246	247-350

Table 9. SC READY and MAP Cut Score Equivalents (Spring): ELA

Grade	SC READY			
	Does Not Meet Expectations	Approaches Expectations	Meets Expectations	Exceeds Expectations
3	100-358	359-451	452-539	540-825
4	100-418	419-508	509-591	592-850
5	100-448	449-556	557-652	653-875
6	100-454	455-574	576-666	667-900
7	100-511	512-614	615-703	704-925
8	100-536	537-641	642-736	737-950
	NWEA MAP			
	Does Not Meet Expectations	**Approaches Expectations**	**Meets Expectations**	**Exceeds Expectations**
3	100-185	186-197	198-207	208-350
4	100-193	193-204	205-212	213-350
5	100-198	199-210	211-219	220-350
6	100-200	201-213	214-222	223-350
7	100-205	206-217	218-226	227-350
8	100-208	209-220	221-229	230-350

The bootstrap standard errors of each equivalent MAP cut score are listed in Tables 10 and 11 for Mathematics and ELA, respectively. They are relatively smalls across all linking studies conducted across grades 3-8, test subjects, and performance levels. This gives us evidence supporting the accuracy of the linking results. However, it is also important to keep in mind that linking is a statistical procedure to estimate the equivalence between two sets of test scores and, therefore, linking results contain estimation error.

Table 10. Equivalent MAP Cut Score (Spring) Bootstrap Standard Errors: Mathematics

Grade	NWEA MAP Scores Reaching Performance Level...					
	Approaches Expectations		Meets Expectations		Exceeds Expectations	
	Cut Score	S.E.	Cut Score	S.E.	Cut Score	S.E.
3	190	0.34	200	0.24	211	0.22
4	201	0.31	211	0.24	219	0.24
5	204	0.38	219	0.26	231	0.27
6	210	0.31	225	0.31	236	0.32
7	216	0.37	232	0.35	242	0.36
8	221	0.32	237	0.33	247	0.37

Table 11. Equivalent MAP Cut Score (Spring) Bootstrap Standard Errors: ELA

Grade	NWEA MAP Scores Reaching Performance Level...					
	Approaches Expectations		Meets Expectations		Exceeds Expectations	
	Cut Score	S.E.	Cut Score	S.E.	Cut Score	S.E
3	186	0.44	198	0.29	208	0.25
4	193	0.44	205	0.28	213	0.24
5	199	0.43	211	0.26	220	0.23
6	201	0.38	214	0.25	223	0.23
7	206	0.35	218	0.25	227	0.23
8	209	0.34	221	0.24	230	0.23

The section above summarizes the linking results from the spring term. Linked MAP test scores were also extended from the spring to the fall and winter terms for the scores reaching performance level "Meets Expectations." These scores are summarized in Table 12. Note that these linked scores were calculated based on the mean MAP scores within each term among all SC students who took the MAP test. Therefore, they reflect expected score equivalents on average among these students and thereby should not be interpreted as accurate estimations for every individual student. The estimation errors around the fall and the winter scores will be larger than those around the spring scores.

Table 12. MAP Cut Score Equivalents

Grade	Mathematics			ELA		
	Fall	Winter	Spring	Fall	Winter	Spring
3	187	194	200	187	194	198
4	200	206	211	197	202	205
5	210	215	219	204	209	211
6	218	222	225	210	212	214
7	226	229	232	214	216	218
8	231	234	237	217	219	221

Classification Accuracy

Table 13 summarizes results from the classification accuracy statistics described in Table 1. These are diagnostics used to evaluate the accuracy of using the NWEA MAP test scores to classify students as proficient (Meets Expectations and Exceeds Expectations) or not proficient (Does Not Meet Expectations and Approaches Expectations) on the SC READY Mathematics and ELA summative assessments. The overall classification accuracy statistics range from 0.85 to 0.90, and the AUC statistics are above 0.92 at all grade levels. These diagnostics provide convincing evidence of good classification accuracy for using the linked MAP scores to estimate students' proficiency status on the SC READY assessments at grades 3-8.

Table 13. Classification Accuracy Results

Grade	Overall Classification Accuracy	False Positive Rate	False Negative Rate	Sensitivity	Specificity	Precision	AUC
Mathematics							
3	0.87	0.18	0.08	0.92	0.82	0.85	0.94
4	0.87	0.16	0.09	0.91	0.84	0.84	0.95
5	0.86	0.16	0.12	0.88	0.84	0.82	0.93
6	0.88	0.10	0.15	0.85	0.90	0.84	0.95
7	0.90	0.09	0.12	0.88	0.91	0.82	0.95
8	0.88	0.10	0.18	0.82	0.90	0.81	0.93
ELA							
3	0.87	0.14	0.13	0.87	0.86	0.87	0.94
4	0.86	0.16	0.12	0.88	0.84	0.87	0.94
5	0.86	0.19	0.10	0.90	0.81	0.85	0.93
6	0.86	0.18	0.11	0.89	0.82	0.84	0.93
7	0.85	0.16	0.14	0.86	0.84	0.84	0.93
8	0.85	0.17	0.14	0.86	0.83	0.85	0.92

CONCLUSIONS

It is important to note that equipercentile linking is a statistical procedure used to facilitate interpretation of scores on the SC READY Mathematics and ELA assessments and the NWEA MAP Growth Mathematics and Reading assessments. Despite good classification accuracy results from this study, there are still important notes of caution to call out in interpreting and using the linked scores.

First, the two tests are constructed differently with regard to test content specifications, test design, and test purpose. For example, the MAP Growth Reading assessment is one of two MAP assessments used to assess students' ELA skills (Language Usage is the other assessment), and focuses on "reading comprehension, understanding of genres and text, and vocabulary" (NWEA, 2019, p.11). The SC READY ELA assessment is composed of two subtests—writing and reading—and measures student performance on Reading – Literary Text, Reading – Informational Text, Inquiry, and Writing (SCDE, 2022). The statistical adjustments in linking do not adjust for differences in content. Therefore, scores on the SC READY and NWEA MAP assessments should not be used interchangeably. The linked scores facilitate comparisons of proficiency status between two assessments, but do not imply equivalence.

Second, while there is a high level of confidence associated with the models, the linked scores are based on a 50% likelihood estimation. This means that not all students who reach a proficiency cut score on MAP will necessarily reach the associated score on SC READY. For example, as we saw in Figure 1 above, while the SC READY 452 cut score for "Meets Expectations" in grade 3 corresponds to the MAP Reading score of 198 on average, there is a wide range of MAP scores among students who reached a 452 on SC READY. The interpretation of the estimated 198 MAP Reading score is that 3rd grade students with this MAP score have a 50% probability of scoring 452 or higher (i.e., reaching "Meets Expectations") on the SC READY ELA test. The results are more accurate for students on average than as associated with individual students.

REFERENCES

Kolen, M. J., & Brennan, R. L. (2014). *Test equating, scaling, and linking: Methods and practices* (3rd ed.). Springer Science + Business Media. https://doi.org/10.1007/978-1-4939-0317-7

Holland, P. W., & Thayer, D. T. (1987). *Notes on the use of log-linear models for fitting discrete probability distributions* (Technical Report 87-79). Princeton, NJ: ETS.

Holland, P. W., & Thayer, D. T. (2000). Univariate and bivariate loglinear models for discrete test score distributions. *Journal of Educational and Behavioral Statistics, 25*, 133–183.

NWEA. (2019). *MAP® Growth™ technical report*. Portland, OR: Author.

Rosenbaum, P. R., & Thayer, D. (1987). Smoothing the joint and marginal distributions of scored two-way contingency tables in test equating. *British Journal of Mathematical and Statistical Psychology, 40*, 43–49.

South Carolina Department of Education. (2022). *SC READY and SCPASS Score Report User's Guide: For Use with Spring 2022 Score Reports*. Columbia, SC: Author.